8.5 Use Properties of Trapezoids and Kites

Before You used properties of special parallelograms.

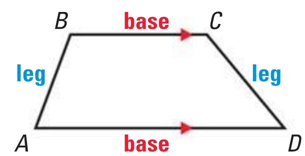
Now You will use properties of trapezoids and kites.

Why? So you can measure part of a building, as in Example 2.

Key Vocabulary
- **trapezoid**
  bases, base angles, legs
- **isosceles trapezoid**
- **midsegment of a trapezoid**
- **kite**

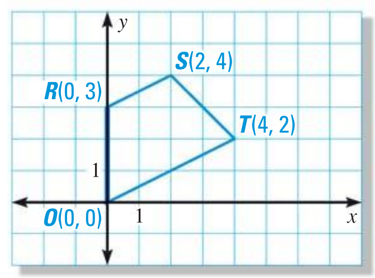
A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. The parallel sides are the **bases**.

A trapezoid has two pairs of **base angles**. For example, in trapezoid $ABCD$, $\angle A$ and $\angle D$ are one pair of base angles, and $\angle B$ and $\angle C$ are the second pair. The nonparallel sides are the **legs** of the trapezoid.



EXAMPLE 1 Use a coordinate plane

Show that *ORST* is a trapezoid.



Solution

Compare the slopes of opposite sides.

Slope of $\overline{RS} = \frac{4-3}{2-0} = \frac{1}{2}$

Slope of $\overline{OT} = \frac{2-0}{4-0} = \frac{2}{4} = \frac{1}{2}$

The slopes of $\overline{RS}$ and $\overline{OT}$ are the same, so $\overline{RS} \parallel \overline{OT}$.

Slope of $\overline{ST} = \frac{2-4}{4-2} = \frac{-2}{2} = -1$

Slope of $\overline{OR} = \frac{3-0}{0-0} = \frac{3}{0}$, which is undefined.

The slopes of $\overline{ST}$ and $\overline{OR}$ are not the same, so $\overline{ST}$ is not parallel to $\overline{OR}$.

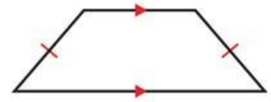
▶ Because quadrilateral *ORST* has exactly one pair of parallel sides, it is a trapezoid.

✓ GUIDED PRACTICE for Example 1

1. **WHAT IF?** In Example 1, suppose the coordinates of point S are (4, 5). What type of quadrilateral is *ORST*? *Explain*.

2. In Example 1, which of the interior angles of quadrilateral *ORST* are supplementary angles? *Explain* your reasoning.

ISOSCELES TRAPEZOIDS If the legs of a trapezoid are congruent, then the trapezoid is an **isosceles trapezoid**.



isosceles trapezoid

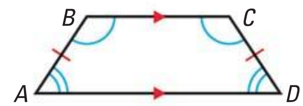
THEOREMS *For Your Notebook*

THEOREM 8.14

If a trapezoid is isosceles, then each pair of base angles is congruent.

If trapezoid $ABCD$ is isosceles, then $\angle A \cong \angle D$ and $\angle B \cong \angle C$.

Proof: Ex. 37, p. 548

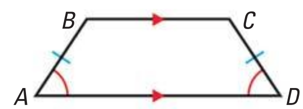


THEOREM 8.15

If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid.

If $\angle A \cong \angle D$ (or if $\angle B \cong \angle C$), then trapezoid $ABCD$ is isosceles.

Proof: Ex. 38, p. 548

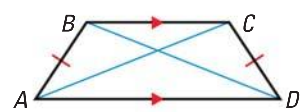


THEOREM 8.16

A trapezoid is isosceles if and only if its diagonals are congruent.

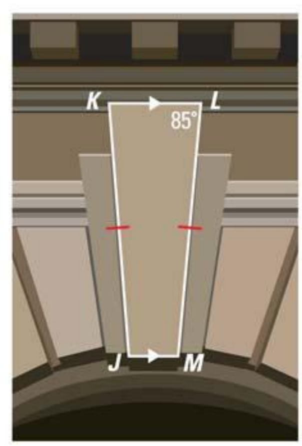
Trapezoid $ABCD$ is isosceles if and only if $\overline{AC} \cong \overline{BD}$.

Proof: Exs. 39 and 43, p. 549



EXAMPLE 2 Use properties of isosceles trapezoids

ARCH The stone above the arch in the diagram is an isosceles trapezoid. Find $m\angle K$, $m\angle M$, and $m\angle J$.



Solution

STEP 1 **Find** $m\angle K$. $JKLM$ is an isosceles trapezoid, so $\angle K$ and $\angle L$ are congruent base angles, and $m\angle K = m\angle L = 85°$.

STEP 2 **Find** $m\angle M$. Because $\angle L$ and $\angle M$ are consecutive interior angles formed by $\overleftrightarrow{LM}$ intersecting two parallel lines, they are supplementary. So, $m\angle M = 180° - 85° = 95°$.

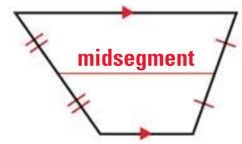
STEP 3 **Find** $m\angle J$. Because $\angle J$ and $\angle M$ are a pair of base angles, they are congruent, and $m\angle J = m\angle M = 95°$.

▶ So, $m\angle J = 95°$, $m\angle K = 85°$, and $m\angle M = 95°$.

READ VOCABULARY

The midsegment of a trapezoid is sometimes called the *median* of the trapezoid.

MIDSEGMENTS Recall that a midsegment of a triangle is a segment that connects the midpoints of two sides of the triangle. The **midsegment of a trapezoid** is the segment that connects the midpoints of its legs.



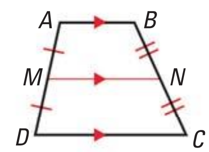
The theorem below is similar to the Midsegment Theorem for Triangles.

THEOREM — *For Your Notebook*

THEOREM 8.17 Midsegment Theorem for Trapezoids

The midsegment of a trapezoid is parallel to each base and its length is one half the sum of the lengths of the bases.

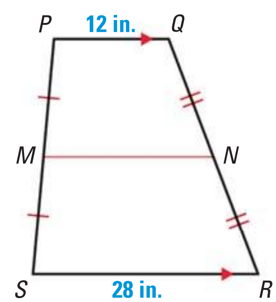
If $\overline{MN}$ is the midsegment of trapezoid *ABCD*, then $\overline{MN} \parallel \overline{AB}$, $\overline{MN} \parallel \overline{DC}$, and $MN = \frac{1}{2}(AB + CD)$.



Justification: Ex. 40, p. 549
Proof: p. 937

EXAMPLE 3 Use the midsegment of a trapezoid

In the diagram, $\overline{MN}$ is the midsegment of trapezoid *PQRS*. Find *MN*.



Solution

Use Theorem 8.17 to find *MN*.

$MN = \frac{1}{2}(PQ + SR)$ — **Apply Theorem 8.17.**

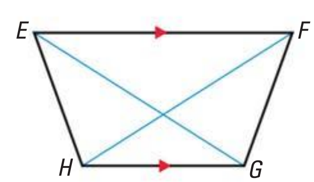
$= \frac{1}{2}(12 + 28)$ — **Substitute 12 for *PQ* and 28 for *XU*.**

$= 20$ — **Simplify.**

▶ The length *MN* is 20 inches.

✓ GUIDED PRACTICE for Examples 2 and 3

In Exercises 3 and 4, use the diagram of trapezoid *EFGH*.

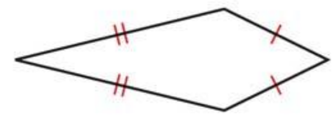


3. If $EG = FH$, is trapezoid *EFGH* isosceles? *Explain*.

4. If $m\angle HEF = 70°$ and $m\angle FGH = 110°$, is trapezoid *EFGH* isosceles? *Explain*.

5. In trapezoid *JKLM*, $\angle J$ and $\angle M$ are right angles, and $JK = 9$ cm. The length of the midsegment $\overline{NP}$ of trapezoid *JKLM* is 12 cm. Sketch trapezoid *JKLM* and its midsegment. Find *ML*. *Explain* your reasoning.

KITES A **kite** is a quadrilateral that has two pairs of consecutive congruent sides, but opposite sides are not congruent.



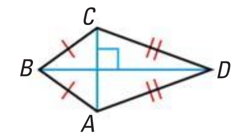
THEOREMS *For Your Notebook*

THEOREM 8.18

If a quadrilateral is a kite, then its diagonals are perpendicular.

If quadrilateral $ABCD$ is a kite, then $\overline{AC} \perp \overline{BD}$.

Proof: Ex. 41, p. 549

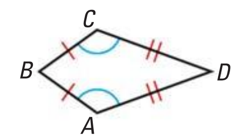


THEOREM 8.19

If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent.

If quadrilateral $ABCD$ is a kite and $\overline{BC} \cong \overline{BA}$, then $\angle A \cong \angle C$ and $\angle B \not\cong \angle D$.

Proof: Ex. 42, p. 549



EXAMPLE 4 Apply Theorem 8.19

Find $m\angle D$ in the kite shown at the right.

Solution

By Theorem 8.19, $DEFG$ has exactly one pair of congruent opposite angles. Because $\angle E \not\cong \angle G$, $\angle D$ and $\angle F$ must be congruent. So, $m\angle D = m\angle F$. Write and solve an equation to find $m\angle D$.

$m\angle D + m\angle F + 124° + 80° = 360°$ — **Corollary to Theorem 8.1**

$m\angle D + m\angle D + 124° + 80° = 360°$ — **Substitute $m\angle D$ for $m\angle F$.**

$2(m\angle D) + 204° = 360°$ — **Combine like terms.**

$m\angle D = 78°$ — **Solve for $m\angle D$.**

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✓ GUIDED PRACTICE for Example 4

6. In a kite, the measures of the angles are $3x°$, 75°, 90°, and 120°. Find the value of x. What are the measures of the angles that are congruent?

8.5 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 11, 19, and 35

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 16, 28, 31, and 36

SKILL PRACTICE

1. **VOCABULARY** In trapezoid $PQRS$, $\overline{PQ} \parallel \overline{RS}$. Sketch $PQRS$ and identify its bases and its legs.

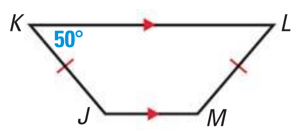
2. ★ **WRITING** *Describe* the differences between a kite and a trapezoid.

EXAMPLES 1 and 2 on pp. 542–543 for Exs. 3–12

COORDINATE PLANE **Points A, B, C, and D are the vertices of a quadrilateral. Determine whether $ABCD$ is a trapezoid.**

3. $A(0, 4)$, $B(4, 4)$, $C(8, -2)$, $D(2, 1)$
4. $A(-5, 0)$, $B(2, 3)$, $C(3, 1)$, $D(-2, -2)$
5. $A(2, 1)$, $B(6, 1)$, $C(3, -3)$, $D(-1, -4)$
6. $A(-3, 3)$, $B(-1, 1)$, $C(1, -4)$, $D(-3, 0)$

FINDING ANGLE MEASURES **Find $m\angle J$, $m\angle L$, and $m\angle M$.**

7.


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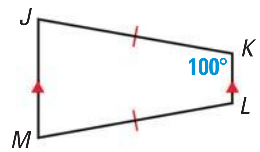

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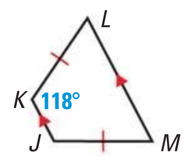
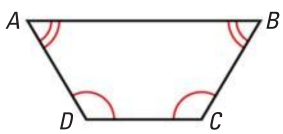


REASONING **Determine whether the quadrilateral is a trapezoid. *Explain*.**

10.


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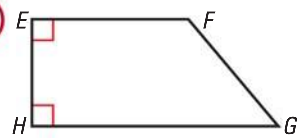
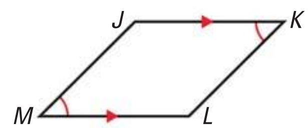
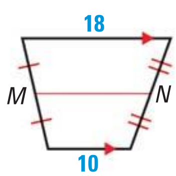
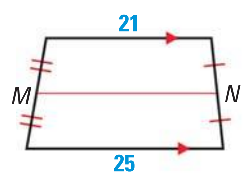


12.


EXAMPLE 3 on p. 544 for Exs. 13–16

FINDING MIDSEGMENTS **Find the length of the midsegment of the trapezoid.**

13.


14.


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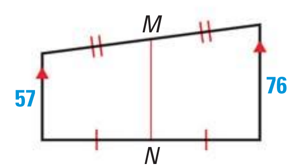


16. ★ **MULTIPLE CHOICE** Which statement is not always true?

Ⓐ The base angles of an isosceles trapezoid are congruent.

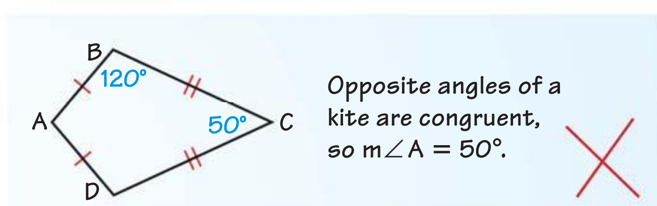
Ⓑ The midsegment of a trapezoid is parallel to the bases.

Ⓒ The bases of a trapezoid are parallel.

Ⓓ The legs of a trapezoid are congruent.

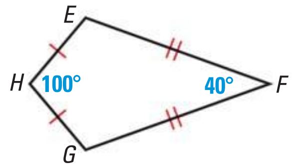
EXAMPLE 4 on p. 545 for Exs. 17–20

17. **ERROR ANALYSIS** *Describe* and correct the error made in finding $m\angle A$.

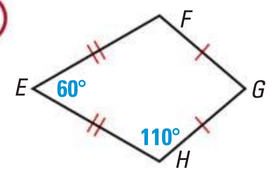


ANGLES OF KITES ***EFGH* is a kite. Find $m\angle G$.**

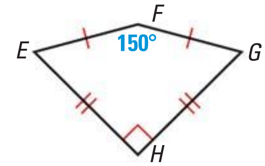
18.



19.

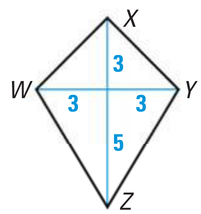


20.

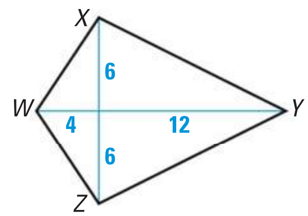


DIAGONALS OF KITES **Use Theorem 8.18 and the Pythagorean Theorem to find the side lengths of the kite. Write the lengths in simplest radical form.**

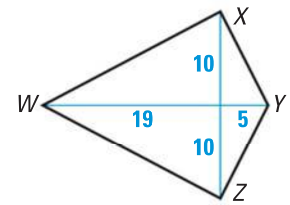
21.



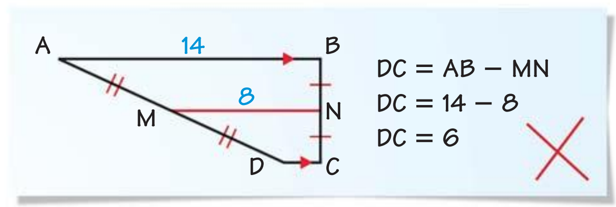
22.



23.

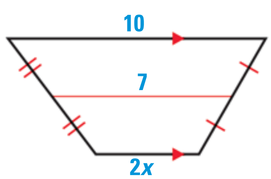


24. ERROR ANALYSIS In trapezoid *ABCD*, $\overline{MN}$ is the midsegment. *Describe* and correct the error made in finding *DC*.

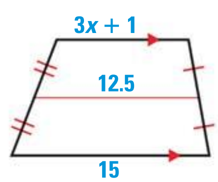


ALGEBRA **Find the value of *x*.**

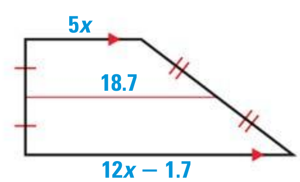
25.



26.



27.



28. ★ SHORT RESPONSE The points $M(-3, 5)$, $N(-1, 5)$, $P(3, -1)$, and $Q(-5, -1)$ form the vertices of a trapezoid. Draw *MNPQ* and find *MP* and *NQ*. What do your results tell you about the trapezoid? *Explain.*

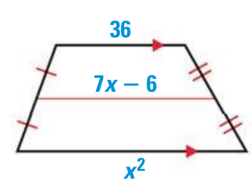
29. DRAWING In trapezoid *JKLM*, $\overline{JK} \parallel \overline{LM}$ and $JK = 17$. The midsegment of *JKLM* is $\overline{XY}$, and $XY = 37$. Sketch *JKLM* and its midsegment. Then find *LM*.

30. RATIOS The ratio of the lengths of the bases of a trapezoid is 1 : 3. The length of the midsegment is 24. Find the lengths of the bases.

31. ★ MULTIPLE CHOICE In trapezoid *PQRS*, $\overline{PQ} \parallel \overline{RS}$ and $\overline{MN}$ is the midsegment of *PQRS*. If $RS = 5 \cdot PQ$, what is the ratio of *MN* to *RS*?

Ⓐ 3 : 5 Ⓑ 5 : 3 Ⓒ 2 : 1 Ⓓ 3 : 1

32. CHALLENGE The figure shown at the right is a trapezoid with its midsegment. Find all the possible values of *x*. What is the length of the midsegment? *Explain.* (The figure may not be drawn to scale.)

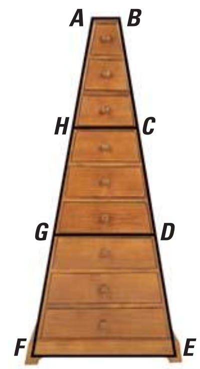


33. REASONING *Explain* why a kite and a general quadrilateral are the only quadrilaterals that can be concave.

PROBLEM SOLVING

EXAMPLES 3 and 4
on pp. 544–545
for Exs. 34–35

34. FURNITURE In the photograph of a chest of drawers, $\overline{HC}$ is the midsegment of trapezoid $ABDG$, $\overline{GD}$ is the midsegment of trapezoid $HCEF$, $AB = 13.9$ centimeters, and $GD = 50.5$ centimeters. Find HC. Then find FE.



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35. GRAPHIC DESIGN You design a logo in the shape of a convex kite. The measure of one angle of the kite is 90°. The measure of another angle is 30°. Sketch a kite that matches this description. Give the measures of all the angles and mark any congruent sides.

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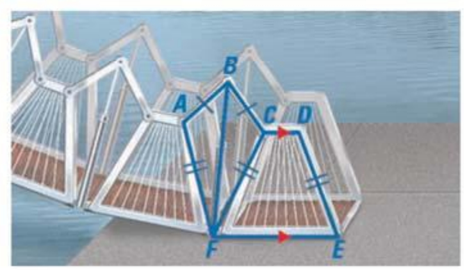
36. ★ EXTENDED RESPONSE The bridge below is designed to fold up into an octagon shape. The diagram shows a section of the bridge.

a. Classify the quadrilaterals shown in the diagram.

b. As the bridge folds up, what happens to the length of $\overline{BF}$? What happens to $m\angle BAF$, $m\angle ABC$, $m\angle BCF$, and $m\angle CFA$?

c. Given $m\angle CFE = 65°$, find $m\angle DEF$, $m\angle FCD$, and $m\angle CDE$. *Explain.*

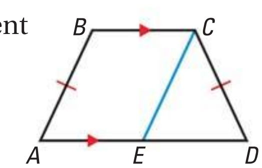


37. PROVING THEOREM 8.14 Use the diagram and the auxiliary segment to prove Theorem 8.14. In the diagram, $\overline{EC}$ is drawn parallel to $\overline{AB}$.

GIVEN ▶ $ABCD$ is an isosceles trapezoid, $\overline{BC} \parallel \overline{AD}$

PROVE ▶ $\angle A \cong \angle D$, $\angle B \cong \angle BCD$

Hint: Find a way to show that $\triangle ECD$ is an isosceles triangle.

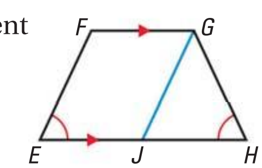


38. PROVING THEOREM 8.15 Use the diagram and the auxiliary segment to prove Theorem 8.15. In the diagram, $\overline{JG}$ is drawn parallel to $\overline{EF}$.

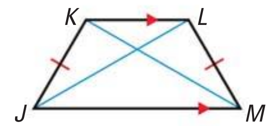
GIVEN ▶ $EFGH$ is a trapezoid, $\overline{FG} \parallel \overline{EH}$, $\angle E \cong \angle H$

PROVE ▶ $EFGH$ is an isosceles trapezoid.

Hint: Find a way to show that $\triangle JGH$ is an isosceles triangle.



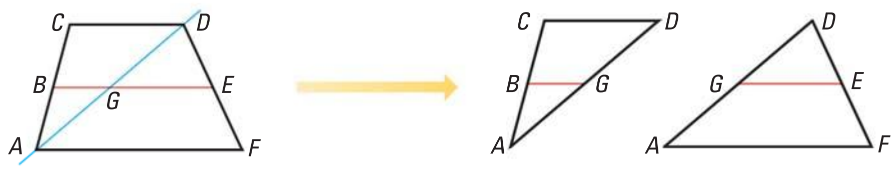
39. PROVING THEOREM 8.16 Prove part of Theorem 8.16.



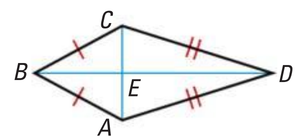
GIVEN ▶ *JKLM* is an isosceles trapezoid.
$\overline{KL} \parallel \overline{JM}$, $\overline{JK} \cong \overline{LM}$

PROVE ▶ $\overline{JL} \cong \overline{KM}$

40. REASONING In the diagram below, $\overline{BG}$ is the midsegment of $\triangle ACD$ and $\overline{GE}$ is the midsegment of $\triangle ADF$. *Explain* why the midsegment of trapezoid *ACDF* is parallel to each base and why its length is one half the sum of the lengths of the bases.



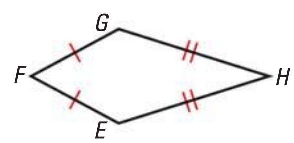
41. PROVING THEOREM 8.18 Prove Theorem 8.18.



GIVEN ▶ *ABCD* is a kite.
$\overline{AB} \cong \overline{CB}$, $\overline{AD} \cong \overline{CD}$

PROVE ▶ $\overline{AC} \perp \overline{BD}$

42. PROVING THEOREM 8.19 Write a paragraph proof of Theorem 8.19.



GIVEN ▶ *EFGH* is a kite.
$\overline{EF} \cong \overline{GF}$, $\overline{EH} \cong \overline{GH}$

PROVE ▶ $\angle E \cong \angle G$, $\angle F \not\cong \angle H$

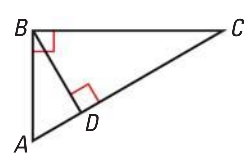
Plan for Proof First show that $\angle E \cong \angle G$. Then use an indirect argument to show that $\angle F \not\cong \angle H$: If $\angle F \cong \angle H$, then *EFGH* is a parallelogram. But opposite sides of a parallelogram are congruent. This result contradicts the definition of a kite.

43. CHALLENGE In Exercise 39, you proved that part of Theorem 8.16 is true. Write the other part of Theorem 8.16 as a conditional statement. Then prove that the statement is true.

MIXED REVIEW

44. Place a right triangle in a coordinate plane in a way that is convenient for finding side lengths. Assign coordinates to each vertex. *(p. 295)*

Use the diagram to complete the proportion. *(p. 449)*



45. $\dfrac{AB}{AC} = \dfrac{?}{AB}$

46. $\dfrac{AB}{BC} = \dfrac{BD}{?}$

PREVIEW
Prepare for Lesson 8.6 in Exs. 47–48.

Three of the vertices of ▱*ABCD* are given. Find the coordinates of point *D*. Show your method. *(p. 522)*

47. $A(-1, -2)$, $B(4, -2)$, $C(6, 2)$, $D(x, y)$

48. $A(1, 4)$, $B(0, 1)$, $C(4, 1)$, $D(x, y)$